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FLOW OF TWO-PHASE MIXTURES IN A ROTARY MIXER

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On the basis of a hydrodynamic model of a multivelocity continuum, the flow of miscible materials over the working surface of a multistage centrifugal (rotary) mixer is investigated, and the optimal dimensions of the working sections required to obtain a high-quality mixture are determined.

In the operation of a rotary mixer, a liquid and a highly disperse solid are supplied to the center of the rotating section of the first stage of the rotor, and layers of the liquid, the forming mixture, and the solid component flow over the surface of a conical channel. Under the action of centrifugal forces, the solid material tends to settle out into the liquid, and then at the edge of the section all this material is dispersed (Fig. 1). The two-phase medium flows on through successive stages of the rotor, which are similar in construction, where the final redistribution of the components occurs.

The motion of each layer of material over the surface of the spinning rotor is described by the equations of fluid mechanics; each layer has its corresponding rheological equation of state. The flow of pure liquid is described by the differential equations

$$\rho_1^0 \frac{d\mathbf{V}_0}{dt} = \rho_1^0 \mathbf{F}_0 + \operatorname{div} \mathbf{T}_0, \tag{1}$$

where ρ_1^0 and $\mathbf{V}_0$ are the density and velocity of the liquid; $\mathbf{T}_0$ is the stress tensor; and $\mathbf{F}_0$ is the mass force.

The mixture which forms constitutes a two-phase medium and can be described on the basis of the multivelocity Rakhmatulin model (if direct collisions and shear strains of the solid particles may be neglected in comparison with the carrier phase) by the equations [1, 2]

$$\frac{\partial \rho_1}{\partial t} + \nabla (\rho_1 \mathbf{V}_1) = 0, \tag{2}$$

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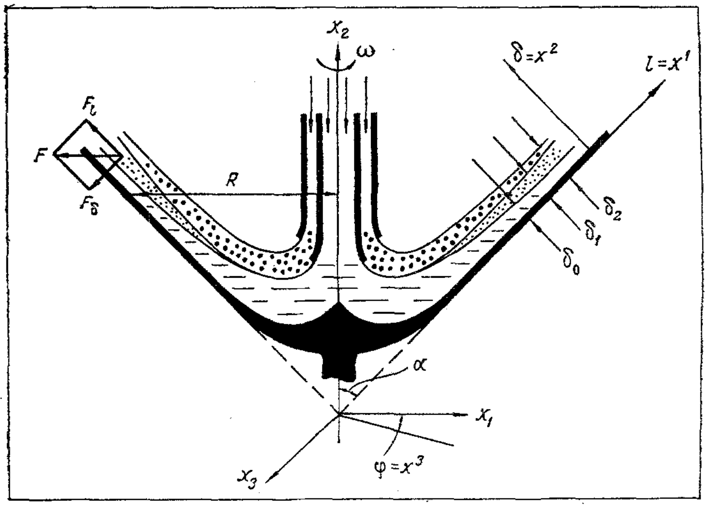

Fig. 1. Flow diagram of miscible materials over working surface of the rotary mixer.

$$\frac{\partial \rho_2}{\partial t} + \nabla (\rho_2 \mathbf{V}_2) = 0, \tag{3}$$

$$\frac{\rho_1 d\mathbf{V}_1}{dt} = -\alpha_1 \nabla \mathbf{P} + \nabla^k \tau_1^k - \mathbf{f}_{12} + \rho_1 \mathbf{F}_1, \tag{4}$$

$$\frac{\rho_2 d\mathbf{V}_2}{dt} = -\alpha_2 \nabla P + \mathbf{f}_{12} + \rho_2 \mathbf{F}_2, \tag{5}$$

$$\tau_1^{ki} = \lambda_1^* \nabla \mathbf{V}_1 + 2\mu_1^* \mathbf{e}^{ki}. \tag{6}$$

The subscripts 1 and 2 denote the liquid and solid phases of the mixture, respectively; λ_1^* and μ_1^* are viscosity coefficients.

The motion of the highly disperse material may be regarded as the motion of a certain continuous medium with a given rheological equation of state. However, the general problem is extremely complicated in this case, and therefore it is assumed that the solid material moves in the longitudinal direction with some averaged velocity $v_e[\delta_1(l)]$, where l varies along the rotor generatrix, and there is no relative motion at the mixture—solid-phase boundary.

Let δ_0 be the thickness of the layer of pure liquid, δ_1 the thickness of the layers of pure liquid and forming mixture, taken together, and δ_2 the thickness of all the layers. The axisymmetric steady flow of miscible materials is considered in the orthogonal axes x^1, x^2, x^3, related to the rotor (Fig. 1), under the following assumptions The effective liquid viscosity is sufficiently large; as a result of the large angular velocity of the rotor, $v_{x^1} \gg v_{x^2}$; the total thickness of the layers is considerably less than the corresponding radius of the conical rotor channel, i. e., $R \gg \delta_2$; and the Coriolis force is negligible in comparison with the centrifugal forces F_l and F_δ, where $l = x^1$, $\delta = x^2$ (Fig. 1).

With these assumptions, the fluid-mechanics equations describing the motion of the liquid and two-phase mixture take the form

$$\frac{1}{R} \cdot \frac{\partial}{\partial x^2} (R \tau_0^{12}) + \rho_1^0 F_{x^1} = 0, \tag{7}$$

$$\frac{1}{R} \cdot \frac{\partial}{\partial x^2} (R \tau_0^{22}) + \rho_1^0 F_{x^2} = 0, \tag{8}$$

$$\frac{\partial}{\partial x^1} (R v_{0x^1}) + \frac{\partial}{\partial x^2} (R v_{0x^2}) = 0, \tag{9}$$

$$\frac{1}{R} \cdot \frac{\partial}{\partial x^2} (R \tau_1^{12}) - f_{12x^1} + \rho_1 F_{x^1} = 0, \tag{10}$$

$$\frac{1}{R} \cdot \frac{\partial}{\partial x^2} (R \tau_1^{22}) - \alpha_1 \frac{\partial P}{\partial x^2} - f_{12x^2} + \rho_1 F_{x^2} = 0, \tag{11}$$

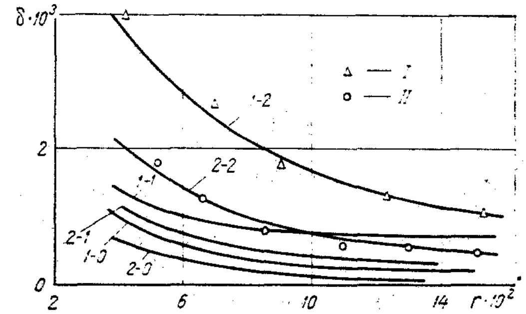

Fig. 2. Comparison of experimental and calculated data on the total thickness of the layers δ_2 and calculated values of δ_1 and δ_0: 1) the system nitropolyether + KCl; $q_1 = 8 \cdot 10^3$ kg/sec; $q_2 = 11 \cdot 10^{-3}$ kg/sec; $\omega = 104.7$ sec^{-1}; 2) glycerin + KCl; $q_1 = 28 \cdot 10^{-3}$ kg/sec; $q_2 = 18.2 \cdot 10^{-3}$ kg/sec; $\omega = 84$ sec^{-1}, where i—2, i—1, i—0 refer, respectively, to δ_2, δ_1, and δ_0 for the i-th system (i = 1, 2). $\delta \cdot 10^3$, m; $r \cdot 10^2$, m.

$$\frac{\partial}{\partial x_1}(\alpha_1 R v_{1x^1}) + \frac{\partial}{\partial x^2}(\alpha_1 R v_{1x^2}) = 0, \tag{12}$$

$$f_{12x^1} + \rho_2 F_{x^1} = 0, \tag{13}$$

$$-\alpha_2 \frac{\partial P}{\partial x^2} + f_{12x^2} + \rho_2 F_{x^2} = 0, \tag{14}$$

$$\frac{\partial}{\partial x^1}(\alpha_2 R v_{2x^1}) + \frac{\partial}{\partial x^2}(\alpha_2 R v_{2x^2}) = 0, \tag{15}$$

$$\alpha_1 + \alpha_2 = 1. \tag{16}$$

Equations (7)-(16) describe the motion of the miscible materials over the rotor surface for any rheological law of state of the medium.

According to the hydrodynamics of a multiphase medium, the phase-interaction force f_{12} in Eqs. (4)-(5) may be written in the first approximation as $f_{12} = f(V_1 - V_2)$, where $f = f(\alpha_2, \eta, d)$ is the phase-interaction coefficient; η is the effective liquid viscosity; and d is the characteristic dimension of the solid particles. Dimensional analysis shows [3] that $[f] = M/L^3T$; $[\eta] = M/LT$; and $[d] = L$ (M, L, and T are the dimensions of mass, length, and time). This leads to the relation

$$f = \frac{\eta}{d^2}\varphi(\alpha_2). \tag{17}$$

It is obvious from physical considerations that $\varphi(0) = 0$, $\varphi(\alpha_2) \to \infty$ as $\alpha_2 \to \alpha_{20}$ (α_{20} is the "maximum-packing" concentration of solid particles). The specific form of the function φ may be found experimentally. The simplest first approximation to the function φ is

$$\varphi(\alpha_2) = A\frac{\alpha_2}{\alpha_{20} - \alpha_2}. \tag{18}$$

Using Eq. (18) in Eq. (17) gives

$$f = A\frac{\eta}{d^2} \cdot \frac{\alpha_2}{\alpha_{20} - \alpha_2}. \tag{19}$$

If the effective liquid viscosity η is large and the characteristic dimension d of the solid particles is small, it follows from Eq. (19) that the coefficient f becomes very large beginning with a certain value of α_2. Hence it folloes that v_1 is little different from v_2. This makes it possible to neglect inertial forces with respect to motion for the solid particles in the equations of motion (13)-(14).

In the equations of motion of the two-phase mixture, the pressure arising due to small-scale perturbations is assumed to be zero, since it is associated with particle collisions in the course of their random motion and is negligible in comparison with the viscosity of the liquid phase.

In motion of the material over the mixer surface, the determining parameters are the velocity of the mixture as a whole along the rotor generatrix v_{x^1}, the rate of settling of the solid particles in the centrifugal field, the layer thicknesses δ_0, δ_1, and δ_2, and the velocities v_{0x^1} and v_{0x^2}. Therefore, adding Eqs. (10)-(15) in pairs gives the equations

$$\frac{1}{R}\cdot\frac{\partial}{\partial x^2}(R\tau^{12})+\rho F_{x^1}=0, \tag{20}$$

$$\frac{1}{R}\cdot\frac{\partial}{\partial x^2}(R\tau^{22})-\frac{\partial P}{\partial x^2}+\rho F_{x^2}=0, \tag{21}$$

$$\frac{\partial R(\alpha_1 v_{1x^1}+\alpha_2 v_{2x^1})}{\partial x^1}+\frac{\partial R(\alpha_1 v_{1x^2}+\alpha_2 v_{2x^2})}{\partial x^2}=0, \tag{22}$$

which, together with Eqs. (7)-(9), are sufficient to determine the desired values v_{0x^1}, v_{0x^2}, and v_{x^1} and describe the motion of the material for any rheological law of state of the liquid and the mixture.

Assume further that the rheological state of the liquid and the relation between the components of the tensors τ^{ki} and $\mathbf{e}^{ki}$ satisfy the rheological power law

$$\mathbf{T}=-P\delta_{ik}+2kI_2^{\frac{n-1}{2}}\mathbf{e}, \tag{23}$$

which holds for many of the systems treated in mixer devices. Here δ_{ik} is the Kronecker delta; $I_2=e^i_k e^k_i$ is a scalar invariant of the strain-rate tensor $\mathbf{e}$. Then the flow properties can be characterized by the equations

$$\begin{gathered}\frac{\partial}{\partial\delta}\left[k\left(\frac{\partial v_{0l}}{\partial\delta}\right)^{n-1}\frac{\partial v_{0l}}{\partial\delta}\right]+\rho_1^0F_l=0,\\ \frac{\partial}{\partial\delta}\left[-P+2k\left(\frac{\partial v_{0l}}{\partial\delta}\right)^{n-1}\frac{\partial v_{0\delta}}{\partial\delta}\right]+\rho_1^0F_\delta=0,\\ \frac{\partial}{\partial\delta}(Rv_{0\delta})+\frac{\partial}{\partial l}(Rv_{0l})=0,\\ \frac{\partial}{\partial\delta}\left[k^*\left(\frac{\partial v_l}{\partial\delta}\right)^{m-1}\frac{\partial v_l}{\partial\delta}\right]+\rho F_l=0,\end{gathered} \tag{24}$$

where k^* is the function giving the concentration of solid phase in the liquid; $F_l=w^2R\sin\alpha$; $F_\delta=-w^2R\cos\alpha$; ρ is the density of the mixture; and k, k^*, n, and m are power-law parameters. After integration of (24) with the following boundary conditions: a) $v_{0l}=v_{0\delta}=0$ for $\delta=0$; b) $v_l=v_{0l}$, $\tau_0^{12}=\tau_0^{22}$ for $\delta=\delta_0$; c) $\partial v_l/\partial\delta=0$ for $0=\delta_1$, we will determine the quantities v_{0l}, $v_{0\delta}$, and v_l:

$$v_{0l}=\left(\frac{n}{n+1}\right)\left(\frac{k}{F_l\rho_1^0}\right)\left\{\left[\frac{\rho F_l(\delta_1-\delta_0)+\rho_1^0F_l\delta_0}{k}\right]^{\frac{n+1}{n}}-\left[\frac{\rho_1^0F_l(\delta_0-\delta)}{k}+\frac{\rho F_l(\delta_1-\delta_0)}{k}\right]^{\frac{n+1}{n}}\right\},$$

$$\begin{aligned}v_{0\delta}l=&-\left(\frac{k}{\rho_1^0F_l}\right)\left[\frac{\rho F_l(\delta_1-\delta_0)+\rho_1^0F_l\delta_0}{k}\right)^{\frac{n+1}{n}}\delta-l\left(\frac{k}{\rho_1^0F_l'}\right)\\&\times\left[\frac{\rho F_l(\delta_1-\delta_0)+\rho_1^0F_l\delta_0}{k}\right]^{\frac{1}{n}}\left[\frac{\rho F_l'(\delta_1'-\delta_0')+\rho_1^0F_l'\delta_1'}{k}\right]\delta\\&+\left(\frac{n}{2n+1}\right)\left(\frac{k}{\rho_1^0F_l}\right)^2\left\{\left[\frac{\rho_1^0F_l\delta_0+\rho F_l(\delta_1-\delta_0)}{k}\right]^{\frac{2n+1}{n}}\right.\\&\left.-\left[\frac{\rho_1^0F_l(\delta_0-\delta)+\rho F_l(\delta_1-\delta_0)}{k}\right]\right\}^{\frac{n+1}{n}}+\left(\frac{n}{n+1}\right)\left(\frac{k}{\rho_1^0F_l'}\right)\left(\frac{k}{\rho_1^0F_l}\right)l\\&\times\left[\frac{\rho_1^0F_l'\delta_0'+\rho F_l'(\delta_1'-\delta_0')}{k}\right]\left\{\left[\frac{\rho_1^0F_l\delta_0+\rho F_l(\delta_1-\delta_0)}{k}\right]^{\frac{n+1}{n}}\right.\end{aligned}$$

$$-\left[\frac{\rho_1^0 F_l(\delta_0-\delta)+\rho F_l(\delta_1-\delta_0)}{k}\right]^{\frac{n+1}{n}}\Bigg\},$$

$$v_l=\left(\frac{m}{m+1}\right)\left(\frac{\rho F_l}{k}\right)^{\frac{1}{m}}\left[(\delta_1-\delta_0)^{\frac{m+1}{m}}-(\delta_1-\delta)^{\frac{m+1}{m}}\right]$$

$$+\left(\frac{n}{n+1}\right)\left(\frac{k}{\rho_1^0 F_l}\right)\Bigg\{\left[\frac{\rho F_l(\delta_1-\delta_0)+\rho_1^0 F_l\delta_0}{k}\right]^{\frac{n+1}{n}}$$

$$-\left[\frac{\rho F_l(\delta_1-\delta_0)}{k}\right]^{\frac{n+1}{n}}\Bigg\},$$

where δ_0', δ_1', and F_l' are derivatives with respect to l.

The unknowns δ_0, δ_1, and δ_2 were determined using the condition of constant flow of the solid (q_1) and liquid (q_2) components and the mechanism of change in the thickness δ_0:

$$\int_0^{\delta_0} 2\pi r\rho_1^0 v_{0l}\,d\delta+\int_{\delta_0}^{\delta_1} 2\pi r\alpha_1\rho_1^0 v_l\,d\delta=q_1, \tag{25}$$

$$\int_{\delta_0}^{\delta_1} 2\pi r\rho_2^0\alpha_2 v_l\,d\delta+2\pi r\rho_{20}\,[v_l(\delta_1)]\,(\delta_2-\delta_1)=q_2, \tag{26}$$

$$\frac{d\delta_0}{dl}=\frac{-W+v_{0,\delta}(\delta_0)}{v_{0l}(\delta_0)}, \tag{27}$$

where $W=\dfrac{d^2(\rho_2^0-\rho_1^0)\,w^2 r}{18\eta}\,\Phi_s\exp(-4.3699\alpha_2^2-4.557\alpha_2)\cos\alpha$ is the collective rate of settling of the solid particles in the centrifugal field, which is determined experimentally; Φ_s is a factor determined by the shape of the solid particles; ρ_{20} is the bulk density of the solid phase; and $r = R-\delta\cos\alpha$. Further, Eqs. (25) and (27) were used to obtain an ordinary differential equation of the form $dy/dl = f(l, \delta_0, y)$, where $y = (\delta_1-\delta_0)(\rho/\rho_1^0)$. This equation and Eq. (27) were solved simultaneously by the Runge–Kutta method, and δ_2 was determined from Eq. (26). Comparison of the calculated and experimental data showed good agreement (Fig. 2: the continuous lines show calculated data, while I and II correspond to experimental values of δ_2); the maximum discrepancy does not exceed 18-20%. By solving this hydrodynamic problem, the length of the generatrix of the first rotor stage which is optimal for high-quality mixing can be found for one of the following conditions: a) $\delta_0 = 0$; b) $\delta_2-\delta_1 = 0$; c) δ_0, $\delta_1-\delta_0$, and $\delta_2-\delta_1$ are of the same order as the dimensions of the agglomerates of solid particles. Calculations for conditions a), b), and c) showed that mixtures of highly disperse materials and viscous liquids are best prepared in multicascade rotors with an extended first stage.

The motion (flow) of the two-phase mixture in the remaining stages of the rotor may be described by Eqs. (10)-(16). It is necessary to determine the longitudinal component of the mixture velocity v_l and the total thickness of the layers δ_2, which are important operating characteristics of the mixer.

From Eqs. (13) and (20) and the condition of constant phase flow rates

$$\frac{\partial}{\partial\delta}\left[k_1^*\left(\frac{\partial v_l}{\partial\delta}\right)^{m_1-1}\frac{\partial v_l}{\partial\delta}\right]+\rho F_l=0, \tag{28}$$

$$\rho_2 F_l+A\,\frac{k}{d^2}\,\frac{\alpha_2}{\alpha_{20}-\alpha_2}\,(v_{1l}-v_{2l})=0, \tag{29}$$

$$\int_0^{\delta_2} 2\pi r\alpha_1\rho_1^0 v_{1l}\,d\delta+\int_0^{\delta_2} 2\pi r\rho_2^0\alpha_2 v_{2l}\,d\delta=q_1+q_2, \tag{30}$$

$$\int_0^{\delta_2} 2\pi r\rho_1^0\alpha_1 v_{1l}\,d\delta=q_1. \tag{31}$$

Integrating Eq. (28) with the boundary conditions $\partial v_l/\partial\delta = 0$ when $\delta = \delta_2$ and $v_l = 0$ when $\delta = 0$ leads to an expression for the velocity v_l,

$$v_l=\left(\frac{m_1}{m_1+1}\right)\left(\frac{k_1^*}{\rho F_l}\right)\Bigg\{\left[\frac{\rho F_l\delta_2}{k_1^*}\right]^{\frac{m_1+1}{m_1}}-\left[\frac{\rho F_l(\delta_2-\delta)}{k_1^*}\right]^{\frac{m_1+1}{m_1}}\Bigg\}. \tag{32}$$

Further, Eqs. (29)-(31) give

$$(\rho_1^0\alpha_1+\rho_2^0\alpha_2)\left(\frac{m_1}{m_1+1}\right)\left(\frac{\rho F_l}{k_1^*}\right)^{\frac{1}{m_1}}\delta_2^{\frac{2m_1+1}{m_1}}+\frac{d^2(\alpha_{20}-\alpha_2)\rho_2^{0\,2}\alpha_2 F_l}{Ak}\delta_2=\frac{q_1+q_2}{2\pi r}, \tag{33}$$

$$(\rho_1^0\alpha_1)\left(\frac{m_1}{2m_1+1}\right)\left(\frac{\rho F_l}{k_1^*}\right)^{\frac{1}{m_1}}\delta_2^{\frac{2m_1+1}{m_1}}=\frac{q_1}{2\pi r}, \tag{34}$$

from which $\delta_2(l)$ and $\alpha_2(l)$ are determined.

Note that if pure liquid flows over the rotor surface, Eq. (32) becomes the solution obtained in [4].

It is also possible to use the Rakhmatulin interpenetration model, together with experimental data, to calculate the flow of materials in other mixers, centrifuges, centrifugal diffuser-atomizers, etc.

NOTATION

V_j, ρ_j, α_j, velocity, mean density, and concentration (by volume) of the j-th phase; ρ_j^0, true density of the j-th phase; **T**, liquid stress tensor; F_j, mass force acting on the j-th phase; τ_1^{ki}, and e^{ki}, stress and strain-rate tensors; f_{12}, phase-interaction force; P, pressure; R, radius of conical rotor channel; x^i, orthogonal coordinates; ρ, V, density and velocity of mixture; η, effective liquid viscosity; d, characteristic dimension of solid particles; ω, angular velocity of rotor; k, k*, n, m, m_1, k_1^*, power-law parameters for liquid and mixture; α, semivertex angle of conical channel; W, collective rate of settling of solid particles; Φ_S, factor determined by the shape of the solid particles; q_j, mass flow rate of j-th phase; $r = R-\delta\cos\alpha$, distance from axis of rotor rotation to an arbitrary point; ρ_{20}, bulk density of solid phase.

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CHARACTERISTICS OF FLOW BETWEEN A ROTATING AND A STATIC DISK IN THE PRESENCE OF RADIAL FLOW

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An improved method is proposed for the calculation of the flow in the gap between a rotating and a static disk in the presence of radial flow. The algorithm of the solution is realized on a Nairi-2 computer.

To solve a number of problems associated with the hydraulic circulation section of a multistage turbine with disk rotors and, in particular, to calculate the axial forces and temperature state of the rotors of a steam turbine, it is necessary to know the radial distribution of the pressure of the medium in the gap between a rotating disk and the corresponding static element (diaphragm, casing). An approximate solution of this problem was obtained in [1] and subsequently refined in [2-4]. In [5], there was further development of the method of calculating the pressure distribution along the disk radius in the presence of radial flow, but the

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